Annals of Nuclear Energy 128 (2019) 380–389

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

A stochastic differential equation for neutron count with detector dead time and applications to the Feynman-α formula

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ARTICLE INFO

Article history:
Received 8 March 2018
Received in revised form 2 December 2018
Accepted 11 January 2019

Keywords:
Reactor noise
Stochastic differential equations
Dead time
Feynman-α method

ABSTRACT

Detector dead time, caused by both physical components in the detection system and the electronic data acquisition, may have a dramatic effect on the regulation system and in-pile experiments. For example, when conducting the Feynman-α experiments in a marginally sub-critical configuration, the dead time effect is known to bias the variance to mean ratio. Analytic computations of the influence of the dead time on the detection count distribution are hard. Therefore, conducting Feynman-α experiments, or other noise experiments, in the presence of a noticeable dead time effect, is challenging. In the present study, we develop the stochastic differential equations approach to stochastic transport, by providing a model for the detection count in a sub-critical configuration under a non-paralyzing detector dead time.

The analysis is based on tools from renewal processes and on a nonlinear filter for detection losses. After constructing the full model, a second order approximation is provided and solved, suggesting a novel first order correction to the Feynman-Y function. The proposed correction is compared with experimental results and past known results, showing improvement and high accuracy.

1. Introduction

Detector dead time losses, caused by both physical components in the detection system and the electronic data acquisition, are perhaps the most prominent effect in non-ideal detector behavior. As the detection rate grows, dead time has a dramatic effect on the regulation system and in-pile experiments (Muller, 1973). In particular, when conducting the Feynman-α experiments, since the dead time has a stronger effect on correlated neutrons, the dead time biases the variance to mean ratio by reducing it (Hashimoto et al., 1996).

The importance of the dead time effect on reactor experiments and radiation measurements in general is widely recognized, and has enjoyed vast treatment since the early 50's of the previous century (a recent review was given in Usman and Patil (2018)). However, in the context of reactor noise, most classic analytic results are inapplicable, since they assume an exponentially distributed waiting time between consecutive detections (which is not the case in noise experiments).

The most standard correction used in practice consists of introducing a simple offset of the variance to mean ratio, given as twice the counts in the dead time gate, as suggested in Hashimoto et al. (1996). This standard correction is known to be precise at the limit $T \to 0$, where the deviation from an exponential waiting time drops to 0 (here T is the length of the detection window), but not for the entire range of values of T (which is typically $0 < T \leqslant 10^{-3}$, Uhrig, 1970).

The influence of the dead time on the detection count distribution in the special context on the Feynman-Y (or variance to mean) formula is also well studied topic, with treatment varying from full first principle modeling (Kitamura and Fukushima, 2014) to experimental numeric corrections (Gilad et al., 2018). However, since the phenomenon has a strong non-linearity, most existing analytic models are either restricted in their parameter range (in terms of the reactivity, count rate, dead time losses), or simply too complicated to solve. Consequently, it is safe to state that a full analytic formula describing the effect of dead time on the Feynman-Y formula is not known, and implementation of noise experiments under a non negligible dead time is bound to create a systematic error.

The outline of the present study is to offer a new modeling scheme for dead time effect on noise experiment. While we cannot claim that this approach results in a full explicit formula, we will show that under some approximations the model is solvable, and the corrections given are an improvement over the existing formulas, and thus reduce systematic errors.

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https://doi.org/10.1016/j.anucene.2019.01.017
0306-4549/

Modeling reactor noise and the stochastic fluctuations of the neutron population and detection (often referred to as *stochastic transport*) is traditionally performed using the Probability Generating Function (PGF) formalism and the master equation (see Pazsit and Pal, 2008 for an overview on the topic). In the last decade, originating from the work of Hayes and Allen (2005), a new modeling approach for reactor noise has been studied, via Stochastic Differential Equations (SDE). The approach is based on diffusion scale approximations, justified by the very large number of reactions and high reaction rate in a nuclear core, to model the stochasticity of the reactions as if they are driven by a Brownian motion. Whereas the original model introduced in Hayes and Allen (2005) only refers to the neutron population size, in a recent study by the authors Dubi and Atar (2018), the neutron population size was coupled with the detection count, resulting in a system of SDE that accounts for the pair: population and detection. It has been shown in Dubi and Atar (2018) that the model is precise up to the second moment, in the sense that the first and second moments are in complete agreement with the classical results obtained using the PGF formalism. The main advantage of models based on SDE is that they are relatively easy to analyze, often via tools from Ito calculus.

This paper is concerned with the study of the detection count in a sub-critical configuration, under a non-paralyzing detector dead time, via the aforementioned SDE modeling approach. Developing this approach to account for dead time is made possible thanks to a representation that we provide for the conditional distribution of the losses given the detection reactions, as well as tools for Brownian approximations for renewal processes. Our first main result uses these elements in order to derive a new version of the system of SDE for the population and detection in presence of dead time. The second is an analysis of this model aimed at evaluating the first and second moments of the detection count distribution. Specifically, a first order approximation for these moments is developed. It is compared with experimental results.

The paper is organized as follows. Below we provide some basic notation and definitions. In Section 2 we provide some background on the essential topics that are in the study: The dead time phenomenon, the basic SDE model for the neutron count distribution and the Feynman-α method. Section 2.4 is concerned with tools from renewal processes, which play a key role in our analysis. In Section 3, which is the main theoretical contribution of this study, we construct a set of SDE for the detection count distribution in the presence of a non-paralyzing dead time. In Section 4 we solve a first order approximation and validate the results experimentally, and in Section 5 we conclude.

1.1. Notation and definitions

This paper addresses a single energy point model. Under this assumption, the neutron population and detection rate in a sub-critical core subjected to an external source is modeled in terms of five parameters:

1. The fission probability per time unit, denoted by λ_f.
2. The absorption probability per time unit, denoted by λ_a.
3. The detection probability per time unit, denoted by λ_d. In most existing detectors, a neutron must be absorbed in order to be detected. Therefore the detected neutrons form a subset of the absorbed ones. This can be expressed by writing $\lambda_a = \lambda_d + \lambda_\ell$, where λ_ℓ refers to absorptions that do not cause detection (where 'ℓ' is mnemonic for *loss*).
4. The distribution of the number of neutrons emitted in a fission (or the *neutron multiplicity*), denoted by $\{p(\nu)\}_{\nu=0}^{\nu_{max}}$. We will denote by $\overline{\nu}$ and $\overline{\nu^2}$ the first and, resp., second moments of this distribution.
5. The source rate S describing the probability per time unit of a neutron to be released from the external source.

We denote by $\lambda = \lambda_f + \lambda_a$ the total reaction probability per time unit. This parameter can otherwise be characterized as the reciprocal average die-away time of a neutron. Moreover, $p_f = \lambda_f/\lambda$ and $p_a = \lambda_a/\lambda$ give the fission and absorption probabilities, respectively.

The SDE model introduced in Dubi and Atar (2018), and that will be further developed in the present study, couples two stochastic processes: the population size at time t, which we denote by N_t, and the number of detector reactions in the interval $[0, t]$, which we denote by D_t. As mentioned above, the goal of this paper is to extend the SDE model to cover detector dead time. We denote the dead time by τ and make a distinction between *detector reactions* and *counts*: "detector reactions" refers to the total number of reactions at the detector, whereas "counts" refers to the number of detection reactions actually counted. Moreover, by *dead time losses* we refer to those detector reactions that are not counted. We denote by C_t the number of counts in the interval $[0, t]$. Clearly, $C_t \leqslant D_t$, and C_t depends on τ whereas D_t does not.

The following standard notations in nuclear engineering are used. The reactivity is denoted by ρ, the delayed neutron fraction by β, and the generation time by Λ. The Rossi-α coefficient is defined by $\alpha = \frac{\beta-\rho}{\Lambda}$. Notice that this parameter is positive, as the system is sub-critical.

2. Background

2.1. Detection dead time

A detection dead time is defined as a time period after a detection, in which the detection system is non-operational. We use the term "detection system" rather than "detector" because the origin of the dead time might be either in the physical detector or in the electronic registration system. The term "dead time" is a general term for the phenomenon, and in the literature we find two main distinctions between dead time models.

1. Paralyzing versus non-paralyzing: this distinction regards the question of whether a detector reaction that is shielded by an earlier reaction will once again inflict a dead time, extending the overall duration of the dead time period. In the non-paralyzing setting, we assume that a shielded reaction does not inflict a further dead time period, and in the paralyzing setting it does. The term "paralyzing" expresses the fact that if the dead time is extendable, once the power exceeds a certain threshold, any *increase* in the power will *reduce* the measured counts, up until the detection system is totally paralyzed.
2. Constant versus random: most of the analysis in the literature assumes that the duration τ in which the detection system is non-operational following a detector reaction is constant. However, in many cases in practice, the duration might vary randomly. In a mean field approximation, the variability of the dead time is expected to have a very small effect (if at all), but if one is interested in higher moments of the count distribution, the effect may be non-negligible (Pal and Pazsit, 2012).

Since the dead time may have a dramatic effect on the performance of the regulation system and the outcome of physical experiments, dead time corrections have been long studied. In terms of the Counts Per Second (CPS) rate, a classic correction appears in Knoll (2000): denoting by n the theoretical detection rate (as if there is no dead time) and the actual measured count rate by m, the following relations are suggested: for a non paralyzing dead time $m = n(1 - n\tau)$ and for a paralyzing setting $m = ne^{-n\tau}$.

In terms of the second moment, and the effect of a dead time τ on the Feynman-Y curve, there are several noticeable studies in the last two decades. In Yamane and Ito (1996), using a semi-empirical model, the authors predict that as $T \to 0$, the Feynman-Y curve has an offset equal to $2R_0\tau$, where R_0 is detection reaction rate (see Section 4.1). This prediction was validated experimentally in Hashimoto et al. (1996) and is currently one of the standard corrections to the Feynman-Y curve. In a recent study by Kitamura and Fukushima (2014), the early ideas of Degweker (1989) were implemented to the setting of reactor noise. Using the PGF and the master equation formalism, Kitamura has established fairly elegant and applicable corrections for the Feynman-Y curve. In Gilad et al. (2018), a more pragmatic approach was taken, suggesting a dead time correction on the Feynman-Y curve though the *backward extrapolation method* (BEX). Despite vast interest and work on the subject, it is still safe to say that the effect of the dead time on the stochastic transport is not satisfactorily quantified.

2.2. SDE model for the detection count

Modeling and analyzing the stochastic nature of fission chains and the detection count in a sub-critical core, often referred to as *stochastic transport*, is a long studied topic, originating in the seminal work of Feynman (1945). Most stochastic models are based on the PGF formalism, obtained via the Chapman-Kolmogorov equation or the so called master equation (see Pazsit and Pal, 2008 and the references within). In the past decade, originating the work of Hayes and Allen (2005), a new modeling approach for the stochastic transport has been studied, based on SDE and using tools from stochastic calculus. The method adopts a Functional Central Limit Theorem (FCLT) approximation to describe the neutron population as a SDE with two coefficients: the drift coefficient, which also appears in the point rector kinetic ordinary differential equation and defines the dynamic of the mean field population, and a the diffusion coefficient, which governs the Brownian motion (BM) intensity.

Since the SDE model was introduced in Hayes and Allen (2005), the model has been adopted by many contributors, including the following (to state a few): in Ha and Kim (2010), the model was extended to a stochastic PDE, allowing 1D spatial dependence of the neutron population, in Ha and Kim (2011), the reactor transient behavior was studied, in Allen (2013), the doubling time of a sub-critical assembly was studied and in da Silva (2016), numeric solutions to the SDE were studied. Recently, in Dubi and Atar (2018), the authors have coupled the Hayes-Allen equation with an equation that describes the detection process. In its simplest (linear) form, the set of equations introduced in Dubi and Atar (2018) is as follows:

$$\begin{cases} dN_t = -\alpha N_t dt + S dt + \sigma_1 dW_t^{(1)} - \sigma_2 dW_t^{(2)}, \\ dD_t = \lambda_d N_t dt + \sigma_2 dW_t^{(2)}. \end{cases} \tag{1}$$

Here, N_t, D_t, S, λ_d and α are the processes and parameters defined earlier in Section 1.1, $W_t^{(1)}, W_t^{(2)}$ are independent standard BMs associated with the fission chains and the detection process, respectively, and σ_1, σ_2 are the respective diffusion coefficients. These are given by the formulas

$$\sigma_1^2 = \frac{S}{\alpha}\left(\lambda_f + \lambda_\ell + \lambda_f\left(\overline{\nu^2} - 2\overline{\nu}\right)\right) + S, \qquad \sigma_2^2 = \frac{S}{\alpha}\lambda_d.$$

The first equation alone describes an Ornstein-Uhlenbeck (OU) process (Øksendal, 2003). In the sequel we shall use the well-known fact that for an OU process (with $\alpha > 0$, as is always the case under consideration in a sub-critical core), the stationary distribution is normal. Specifically, $N_{\text{stat}} \propto \mathcal{N}(S/\alpha, (\sigma_1^2 + \sigma_2^2)/(2\alpha))$. Moreover, if we denote $\mathcal{C} = \frac{\lambda_f \overline{\nu(\nu-1)}}{2\alpha}$, then the second and third moments of N_{stat} are given by

$$E\left(N_{\text{stat}}^2\right) = \left(\frac{S}{\alpha}\right)^2 + \frac{S}{\alpha}(1 + \mathcal{C}); \quad E\left(N_{\text{stat}}^3\right) = \left(\frac{S}{\alpha}\right)^3 + \left(\frac{S}{\alpha}\right)^2 3(1 + \mathcal{C}) \tag{2}$$

2.3. The Feynman-α method

The Feynman-α method (or Feynman-α experiment) is an in-pile experiment aimed at determining the decay coefficient of a sub-critical core. One of the main appeals of the Feynman-α experiment is the simplicity of the experimental setting and the execution of the experiment: once the neutron population has reached a steady state (the core is subjected to an external source), the detection counts are taken in time stamping mode. Then, for a range of values $T_{min} < T_j < T_{max}$, the detection signal is broken into N_j time gates of duration T_j. For each j we denote by $X_{k,j}$ the number of detection reactions in the k-th gate, and the mean and the variance of the count distribution is estimated by

$$E_j = \sum_{k=1}^{N_j} X_{j,k}; \quad Var_j = \sum_{k=1}^{N_j} (X_{j,k} - E_j)^2.$$

The Feynman-Y curve (or the Feynman variance to mean) is defined by

$$Y_{\text{sampled}}(T_j) = \frac{Var_j}{E_j} - 1, \tag{3}$$

and is fitted to the Feynman-Y function, given by Uhrig (1970)

$$Y(T) = Y_\infty\left(1 - \frac{1 - e^{-\alpha T}}{\alpha T}\right), \tag{4}$$

where α is the Rossi-α. Once the fit is performed and α is estimated, the reactivity can be computed. The model presented above does not incorporate the correlation analysis of the delayed neutrons, which can be translated to a restriction on the time gate $T_{max} \leqslant 0.1$[s] (Uhrig, 1970). The coefficient Y_∞ can be explicitly written in term of the system parameters as $Y_\infty = \frac{\overline{\nu(\nu-1)}\lambda_f \lambda_d}{\alpha^2}$.

2.4. Modeling via renewal processes

The motivation for working here with renewal processes stems from the fact that a straightforward extension of the argument from Dubi and Atar (2018) to cover dead times fails. This is due to the fact that this argument is based on statistical independence of the detection reactions of different neutrons. The dead times certainly create dependence, preventing the use of a simple FCLT approximation to the total count distribution. This applies to estimates of the mean, and even more to estimates of the variance.

The role played by renewal processes in the modeling of neutron counting systems under dead time has been noticed and analyzed previously in Pal and Pazsit (2012) (see also references therein).

Some relevant properties of renewal processes A renewal process counts the number of events occurring in a time interval $[0, t]$, as a function of t, where the waiting times between consecutive events are independent and identically distributed (IID). One may write down an expression for its sample paths in the following way. Consider a sequence of positive IID random variables $\{X_j\}_{j=1}^{\infty}$ and let $J_n = \sum_{k=1}^{n} X_j$ (with $J_0 = 0$). Let

$$R(t) = \sum_{n=1}^{\infty} \mathbb{I}_{\{J_n \leqslant t\}} = \sup\{n \geq 0 : J_n \leqslant t\} \tag{5}$$

where $\mathbb{I}_A$ is the indicator of the event A, defined by $\mathbb{I}_A = 1$ if A holds, 0 otherwise. Then $R(t), t \geq 0$, is the *renewal process associated with the waiting times* $\{X_j\}_{j=1}^{\infty}$, and $J_n, n \geq 1$, are the corresponding *jump times*. Note that X_j is the waiting time between the j-th and the $(j-1)$-th event, and J_n is the time of the n-th event.

Equivalently, if a counting process $R(t)$ describes the number of events in an interval $[0, t]$, and the waiting times between consecutive events is a sequence of IID random variables, then $R(t)$ is a renewal process.

Properties of renewal processes have long been studied, and the theory far exceeds the scope of this paper. The main aspect required in this paper is that they obey the FCLT (Billingsley, 2013).

Theorem 2.1. *Let $X = \{X_j\}_{j=1}^{\infty}$ be a sequence of IID positive random variables with a finite expectation μ_X and finite variance σ_X^2, and denote by $R(t)$ the corresponding renewal process. Then, as $n \to \infty$, the process*

$$\frac{R(nt) - n\mu t}{\sigma\sqrt{n}}$$

converges in law to a standard BM, where $\mu = \mu_X^{-1}$ and $\sigma = \sigma_X \mu_X^{-3/2}$.

Remark 2.2. Whereas the above result is concerned with the convergence of processes, it implies the convergence of random variables, by selecting $t = 1$. In particular, it implies that for large n,

$$R(n) \approx \frac{n}{\mu_X} + \mathcal{N}\left(0, \sigma_X^2 \mu_X^{-3} n\right).$$

Remark 2.3. The special case in which X_j are exponential random variables with parameter λ corresponds to the renewal process being a Poisson process of rate λ. In particular, for any given τ, the random variable $R(\tau)$ has Poisson distribution with parameter $\lambda\tau$.

3. Derivation of SDE

This section contains the first main contribution of this paper, namely the derivation of an SDE model for the count distribution with dead time. The derivation is performed by looking at an interval $[t, t + \Delta t]$, approximating the increment of the dependent variables as a mean field term and a noise term, and taking the $\Delta t \to 0$ limit.

As already mentioned, the analysis in Dubi and Atar (2018) for the case without dead time is concerned with two processes: the neutron population at time t, N_t, and the number of detector reactions in the interval $[0, t], D_t$. In this paper we add one more unknown: the number of counts in the interval $[0, t]$, denoted by C_t. When we wish to emphasize the dependence on τ, we write $C_{t|\tau}$ for C_t.

The section is organized as follows. In Section 3.1 we derive the dynamics of the count distribution given the value of N_t (under the model assumption introduced in Section 1.1). In Section 3.2 we compute the covariance between N_t and C_t, and in Section 3.3 we present the full model which describes the joint dynamics of the two processes.

3.1. Approximating the count distribution under dead time

We analyze the mean value and variance of the number of detector reactions ΔD and counts ΔC in a short interval of duration Δt. As a starting point, the term "short" is used to describe a time interval significantly shorter than the reactor multiplication time $1/\alpha$. In such a short interval we may regard N_t as a fixed quantity. Conditioned on N_t, the waiting time between consecutive detector reactions (at this point detector reactions, not counts!) is a random variable distributed exponentially with parameter $\lambda_d N_t$. In terms of a renewal process, if we denote by $\{X_n\}_{n=1}^{\infty}$ the sequence of waiting times between consecutive detector reactions and assume that a first reaction occurs at time t_0, then $J_n = t_0 + \sum_{j=1}^{n} X_n$ is the time of the $(n+1)$ detector reaction, and the number of detector reactions in the interval $[t_0, t_0 + \Delta t]$ can be interpreted as the renewal process associated with J_n. Thus for $[t_0, t_0 + \Delta t]$,

$$\Delta D = D(t_0 + \Delta t) - D(t_0) = \sum_{n=1}^{\infty} \mathbb{I}_{\{J_n \leqslant t_0 + \Delta t\}}.$$

Now, consider $\left\{\widetilde{J}_n\right\}$ to be the refinement of the sequence $\{J_n\}$ obtained by deleting all values of J_n which are covered by the dead time of a previous detection (see Fig. 1). Then $\widetilde{J}_n$ is the waiting time for the n-th count in the presence of a dead time τ. The renewal process associated with $\widetilde{J}_n$ gives the number of counts within the interval $[t + 0, t_0 + \Delta t]$. Thinking of $\widetilde{J}_n$ as refinement of J_n is very useful, because if the waiting time between consecutive detector reactions has an exponential distribution with parameter $\lambda_d N_t$, then the waiting time starting when the dead time window is complete until the next detector reaction (depicted in Fig. 1 as $t_1, t_2, \ldots, t_4$) is also exponentially distributed with parameter $\lambda_d N$, and these random variables form an IID sequence. This last statement is based on the memoryless property of the exponential random variable.

The above observations show that if the waiting time between consecutive counts is $\widetilde{X}_n = \widetilde{J}_n - \widetilde{J}_{n-1}$, then $\tilde{X}_n$ may be written as $\widetilde{X}_n = \tau + \xi_n$, where ξ_n are IID exponentially distributed with parameter $\lambda_d N_t$. Using the properties of an exponential distribution, the mean value and variance of the waiting time between consecutive counts are given by

$$E\left(\widetilde{X}_n\right) = E(\tau + \xi_n) = \tau + \frac{1}{\lambda_d N_t} \tag{6}$$

$$Var\left(\widetilde{X}_n\right) = Var(\xi_n) = \frac{1}{(\lambda_d N_t)^2}. \tag{7}$$

The total number of counts in the interval $[t_0, t_0 + \Delta t]$ is given as $\Delta C = C(t_0 + \Delta t) - C(t) = \sum_{n=1}^{\infty} \mathbb{I}_{\left\{\widetilde{J}_n \leqslant t_0 + \Delta t\right\}}$. Looking at Eq. (1), the mean field dynamics are governed by two rates: The decay coefficient α and the count rate λ_d. Under the assumption that the count rate is sufficiently faster than any flux transient, which is the case in marginally sub-critical cores, we may assume that ΔC takes the limiting form stated by Theorem 2.1, and using Remark 2.2 we have

$$\Delta C = \frac{\Delta t}{\tau + 1/(\lambda_d N_t)} + \mathcal{N}\left(0, \Delta t \frac{1/(\lambda_d N_t)^2}{(\tau + 1/(\lambda_d N_t))^3}\right) \tag{8}$$

$$= \frac{\lambda_d N_t \Delta t}{1 + \tau \lambda_d N_t} + \mathcal{N}\left(0, \frac{\Delta t \lambda_d N_t}{(1 + \tau \lambda_d N_t)^3}\right). \tag{9}$$

Next, if $\{W_t\}$ is a standard BM and ΔW_t denotes $W_{t+\Delta t} - W_t$, then we can use the fact that ΔW_t is distributed according to $\mathcal{N}(0, \Delta t)$ to write the above equality as

$$\Delta C \stackrel{d}{=} \frac{\lambda_d N_t \Delta t}{1 + \tau \lambda_d N_t} + \sqrt{\frac{\lambda_d N_t}{(1 + \tau \lambda_d N_t)^3}} \Delta W_t. \tag{10}$$

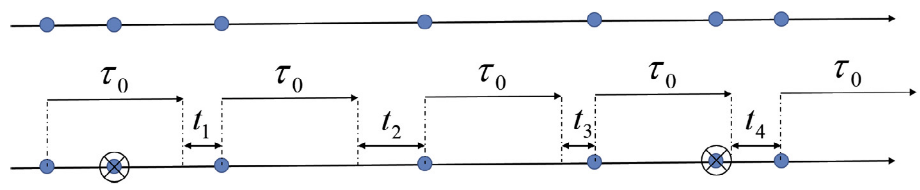

Fig. 1. Detections versus counts as renewal processes.

Table 1
Experimental setting: inflicted dead time τ and the dead time losses.

τ [$\times 10^{-7}$ s^{-1}]	3.25	5.7	8.4	12.6	16.8	25.2
R_c[$\times 10^4$ s^{-1}]	3.86	3.83	3.79	3.77	3.66	3.54
Losses [%]	1.3%	2.2%	3.3%	4.9%	6.5%	9.8%

Table 2
Computed values or R_C using Eq. (33).

τ [$\times 10^{-7}$ s^{-1}]	3.25	5.7	8.4	12.6	16.8	25.2
R_c[$\times 10^4$ s^{-1}]	3.86	3.82	3.78	3.72	3.65	3.52

3.2. Correlation analysis between the count distribution and the neutron population

Eq. (10), combined with the equations derived in Dubi and Atar (2018), provide the following three coupled equations,

$$\begin{cases} \Delta N_t = -\alpha N_t \Delta t + \sigma_1 \Delta W_t^{(1)} - \sigma_2 \Delta W_t^{(2)} + S\Delta t, \\ \Delta D_t = \lambda_d N_t \Delta t + \sigma_2 \Delta W_t^{(2)}, \\ \Delta C_t = \frac{\lambda_d N_t}{1+\tau\lambda_d N_t}\Delta t + \sqrt{\frac{\lambda_d N_t}{(1+\tau\lambda_d N_t)^3}}\Delta W_t^{(3)}. \end{cases} \quad (11)$$

Note that if we are only interested in C_t, the second equation can be removed.

Next, as explained in Dubi and Atar (2018), $W^{(1)}$ is the noise term associated with all reactions except the detections, and consequently, it is independent of $W^{(2)}$ and $W^{(3)}$. However, $\Delta W_t^{(2)}$ and $\Delta W_t^{(3)}$ are correlated, and in order to give a complete description of the model, their correlation must be computed.

To this end, by multiplication of the second and third equations in (11), we have

$$\begin{aligned} E(\Delta D_t \Delta C_t) &= E\left(\left(N\lambda_d \Delta t + \sigma_2 \Delta W_t^{(2)}\right)\left(\frac{\lambda_d N_t \Delta t}{1+\tau\lambda_d N_t} + \sqrt{\frac{\lambda_d N_t}{(1+\tau\lambda_d N_t)^3}}\Delta W_t^{(3)}\right)\right) \\ &= E\left(N\lambda_d \Delta t \frac{\lambda_d N_t \Delta t}{1+\tau\lambda_d N_t}\right) + \sigma_2\sigma_3 E\left(\Delta W_t^{(2)} \Delta W_t^{(3)}\right). \end{aligned} \quad (12)$$

Denote by $\Delta L_t = \Delta D_t - \Delta C_t$ the number of lost counts. We can then write

$$E(\Delta D_t \Delta C_t) = E\left(\Delta C_t^2\right) + E(\Delta L_t \Delta C_t).$$

Conditioning on ΔC_t, we may write $E(\Delta L_t \Delta C_t) = E\{\Delta C_t E(\Delta L_t | \Delta C_t)\}$. By Remark 2.3, the number of detector reactions in an interval of duration τ, conditioned on N_t, has a Poisson distribution with parameter $\lambda_d N_t \tau$. Conditioned on there being ΔC_t counts, the number of lost detector reactions is thus Poisson with parameter $\Delta C_t \lambda_d N_t \tau$. Hence $E(\Delta L_t | \Delta C_t) = \Delta C_t \lambda_d N_t \tau$, and so

$$\begin{aligned} E(\Delta D_t \Delta C_t) = &\ E\left(\Delta C_t^2\right) + \lambda_d N_t \tau E\left(\Delta C_t^2\right) = (1+\lambda_d N_t \tau) E\left(\Delta C_t^2\right) \\ = &\ (1+\lambda_d N_t \tau)\left[\left(\frac{\lambda_d N_t \Delta t}{1+\tau\lambda_d N_t}\right)^2 + \sigma_3^2 E\left(\left(\Delta W_t^{(3)}\right)^2\right)\right]. \end{aligned} \quad (13)$$

Comparing (12) and (13), the term proportional to $(\Delta t)^2$ cancels out, and we have

$$E\left(\Delta W_t^{(2)} \Delta W_t^{(3)}\right) = \frac{1}{\sigma_2\sigma_3}\frac{\lambda_d N_t}{(1+\tau\lambda_d N_t)^2}\Delta t. \quad (14)$$

3.3. The SDE model

We now extend the derivation over $[t, t+\Delta t]$ to an equation over $[0, t]$. By taking the limit $\Delta t \to 0$ we obtain a set of SDE as follows:

$$\begin{cases} dN_t = -\alpha N_t dt + S dt + \sigma_1 dW_t^{(1)} - \sigma_2 dW_t^{(2)}, \\ dC_t = \lambda_d \frac{N_t}{1+\tau\lambda_d N_t} dt + \sigma_3 dW_t^{(3)}, \end{cases} \quad (15)$$

where

$$\sigma_1^2 = \frac{S}{\alpha}\left(\lambda_f + \lambda_\ell + \lambda_f\left(\overline{\nu^2} - 2\overline{\nu}\right)\right) + S; \quad \sigma_2^2 = \lambda_d \frac{S}{\alpha}; \quad \sigma_3^2 = \frac{\lambda_d N_t}{(1+\lambda_d \tau N_t)^3}, \quad (16)$$

and the correlation between $W^{(2)}$ and $W^{(3)}$ is given by

$$dW_t^{(2)} dW_t^{(3)} = \frac{1}{\sigma_2\sigma_3}\frac{\lambda_d N_t}{(1+\tau\lambda_d N_t)^2} dt.$$

Next, as in Dubi and Atar (2018), we make the approximation that the diffusion coefficients are constant, replacing their dependence on N_t by the steady state solution N_{stat}. This is a reasonable assumption when the system has reached its steady state, since the power fluctuation are much smaller than the power itself (and the square root is taken). Yet, this also restricts the validity of the model to steady state analysis, and makes all formulas hereon inapplicable for non stationary scenarios such as power transients, rod drops or polsed source.

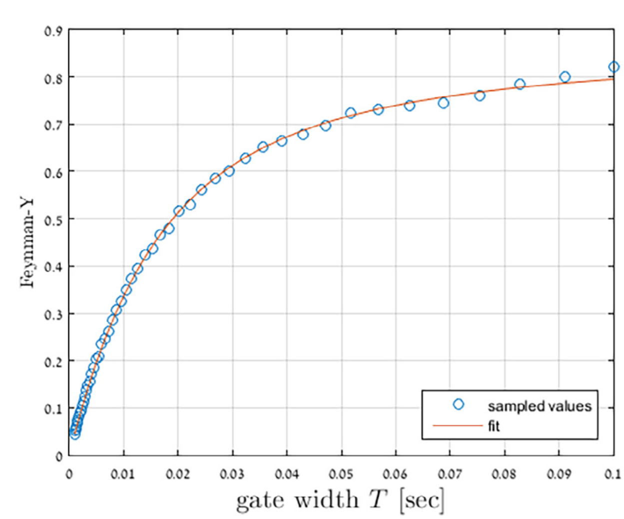

Fig. 2. Sampled values of the Feynman-*Y* curve, together with the fitted function.

The above approximation results in a set of equations

$$\begin{cases} dN_t = -\alpha N_t dt + S dt + \sigma_1 dW_t^{(1)} - \sigma_2 dW_t^{(2)}, \\ dC_t = \lambda_d \frac{N_t}{1+\tau\lambda_d N_t} dt + \sigma_3 dW_t^{(3)}, \end{cases} \tag{17}$$

where

$$\sigma_3^2 = E\left(\frac{\lambda_d N_{\text{stat}}}{(1+\lambda_d \tau N_{\text{stat}})^3}\right)$$

and $W^{(2)}$ and $W^{(3)}$ are correlated via $dW_t^{(2)} dW_t^{(3)} = \tilde{\rho} dt$, where

$$\tilde{\rho} = \frac{1}{\sigma_2 \sigma_3} E\left(\frac{\lambda_d N_{\text{stat}}}{(1+\lambda_d \tau N_{\text{stat}})^2}\right).$$

Eq. (17) is the main theoretical result of this paper. As we later demonstrate, this model can prove very useful and has been successful in predicting the dead time effect on the second moment of the neutron count distribution.

Remark 3.1. Although the set of equations is non-linear, the first equation, which is autonomous, is linear. Its stationary distribution exists and is normal $N_{\text{stat}} \propto \mathcal{N}(S/\alpha, (\sigma_1^2 + \sigma_2^2)/(2\alpha))$. Thus, σ_3 and $\tilde{\rho}$ are defined as the average of a rational function of a normally distributed random variables. Even though the average does not have an explicit formula, it may be computed numerically to any accuracy.

Remark 3.2. The transition from Eq. (11), dealing with a finite interval, to the SDE (15) and then (17) involves taking the limit $\Delta t \to 0$, which seems to stand in contradiction to the assumption $\Delta t > \tau$. This can be justified since the proposed model does not analytically correct the count distribution, but rather emulates the effect of the dead by a non linear filter on the detection rate. Since the correction is done on a dynamic setting (in contract to the classic correction in Knoll (2000), which has the exact same form, but assumes a constant detection rate), this correction is able to account for the entire distribution and not only the mean value. In a sense, this is the best one can hope for in this setting: as a model based on SDE, it is Markovian. The count distribution, on the other hand, is not, as the system must hold memory accounting for the arrival time of the last detection in order to determine when to recuperate. Therefore a local approximation must be applied.

4. Quadratic approximation for small count losses

Eq. (17) is hard to fully analyze. One can write down an equation for the probability distribution function using the Fokker–Planck equation, but explicit expressions for the first and second moment cannot be easily obtained.

The goal of this section is to compute an approximation for the mean and variance on the detection count process for τ small (with respect to the reciprocal of the total count rate).The first order approximation corresponds to a quadratic approximation to Eq. (17). Since the equations are nonlinear, the solution for the detection count process is not normally distributed. However, using Ito's formula, we can explicitly solve the approximation. Based on this approximation, we will also derive a first order correction to the Feynman-Y formula.

The quadratic approximation is simply achieved by removing all higher powers of the dead time τ from the equations, which naturally requires τ to be small. However, τ is a dimensional variable (with units of time), and the term "small" is meaningful only when used with respect to a natural quantity of the model, mea-

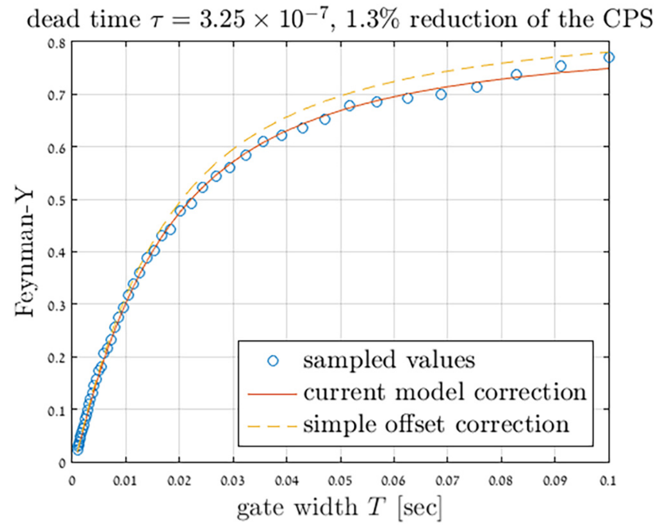

Fig. 3. Dead time correction on the Feynman-Y plot for experiment No. 1.

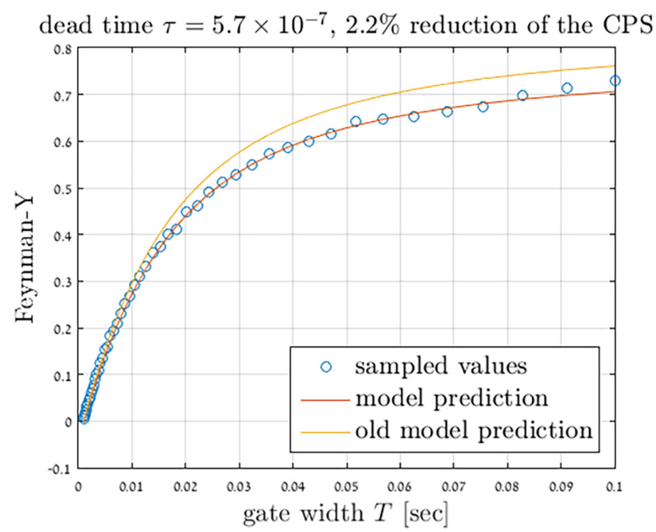

Fig. 4. Dead time correction on the Feynman-Y plot for experiment No. 2.

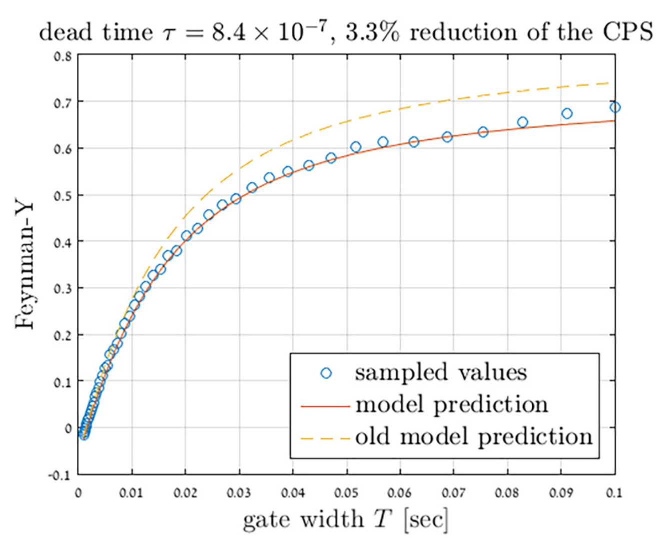

Fig. 5. Dead time correction on the Feynman-Y plot for experiment No. 3.

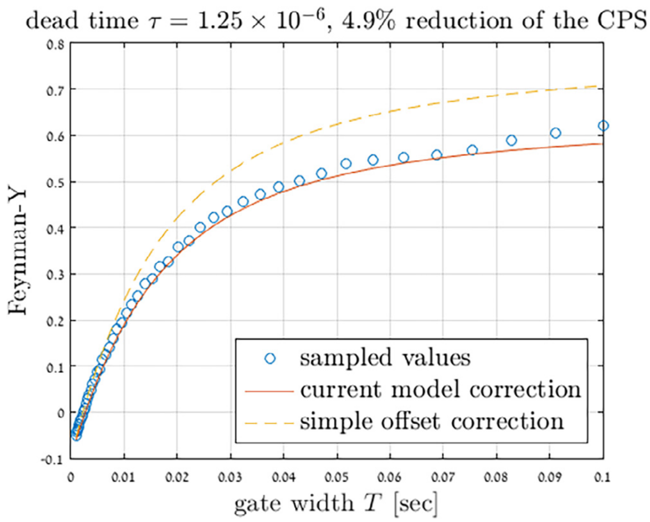

Fig. 6. Dead time correction on the Feynman-*Y* plot for experiment No. 4.

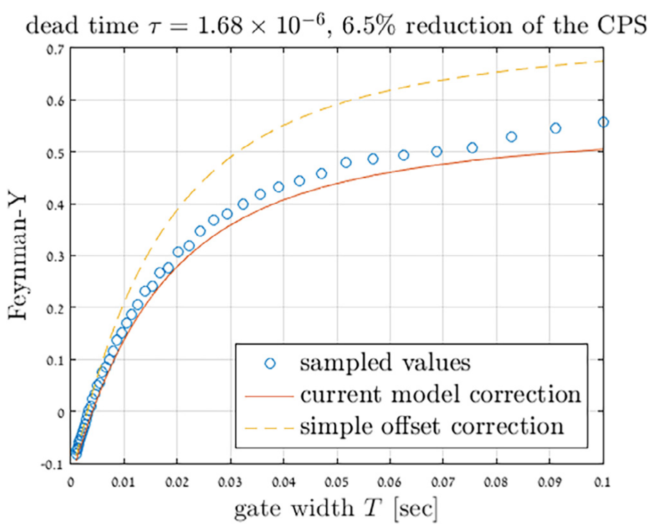

Fig. 7. Dead time correction on the Feynman-*Y* plot for experiment No. 5.

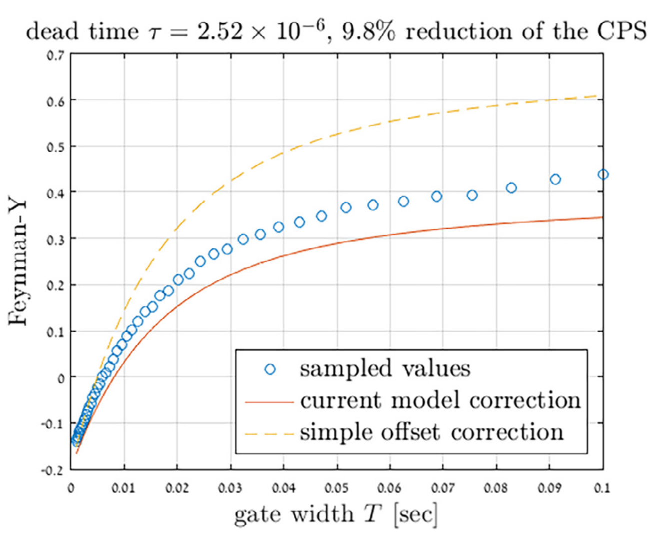

Fig. 8. Dead time correction on the Feynman-*Y* plot for experiment No. 6.

sured in time units. Looking at Eq. (17), we see that the dead time τ is always multiplied by the detection rate λ_d and the population size N_t or N_{stat}. As a result, the accuracy of the approximation demands that the unit free variable $R_0\tau$ will be small (where R_0 is defined in Section 2.1 and Section 4.1). Since, as we will later see, $R_0\tau$ approximate the relative count loss, as a rule of thumb, we expect the quadratic approximation to be reasonable for systems were the dead time losses are less than 10%, restricting the biasing due to the linear approximation in roughly 1%. In what follows, then, we assume that the count loss is relatively small.

4.1. Integral formula for the first two moments of the count distribution

Using the approximation $(1+\delta)^n = 1 - n\delta + o(\delta)$ for small δ, we approximate (17) by

$$\begin{cases} dN_t = -\alpha N_t dt + S dt + \sigma_1 dW_t^{(1)} - \sigma_2 dW_t^{(2)}, \\ dC_t = \lambda_d N_t (1 - \tau\lambda_d N_t) dt + \sigma_3 dW_t^{(3)}. \end{cases} \tag{18}$$

where σ_1, σ_2 are as before, but now

$$\sigma_3^2 = E[\lambda_d N_{\text{stat}} (1 - 3\lambda_d \tau N_{\text{stat}})] \tag{19}$$

and

$$\tilde{\rho} = \frac{1}{\sigma_2\sigma_3} E[\lambda_d N_{\text{stat}} (1 - 2\lambda_d \tau N_{\text{stat}})]. \tag{20}$$

We next aim at computing $E[C_t]$ and $E\left[C_t^2\right]$. To this end, write

$$\begin{aligned} E(C_t) = &\ E\left(\int_0^T \lambda_d N_t (1 - \tau\lambda_d N_t) dt\right) \\ = &\ \lambda_d \int_0^T E(N_t) dt - \lambda_d^2 \tau \int_0^T E\left(N_t^2\right) dt \\ = &\ \left(\lambda_d \tfrac{S}{\alpha} - \tau\lambda_d^2 \left(\left(\tfrac{S}{\alpha}\right)^2 + \tfrac{S}{\alpha}(\mathcal{C}+1)\right)\right) T. \end{aligned} \tag{21}$$

Since the number of counts is linear with T, the term $R_C := \left(\lambda_d \frac{S}{\alpha} - \tau\lambda_d\left(\left(\frac{S}{\alpha}\right)^2 + \frac{S}{\alpha}(\mathcal{C}+1)\right)\right)$ is interpreted as the count rate with dead time τ. If the dead time is nullified, that is $\tau = 0$, then the count rate reduces to the detection reaction rate R_0, given by $R_0 := \lambda_d \frac{S}{\alpha}$.

Computing the second moment requires several steps. First, by Ito's formula (Øksendal, 2003),

$$C_T^2 = C_0 + 2\int_0^T C_s dC_s + \int_0^t (dC_s)^2, \tag{22}$$

where $(dC_t)^2$ is given by $\sigma_3^2 dt$. Taking expectation, noting that $C_0 = 0$, gives

$$\begin{aligned} E\left(C_T^2\right) = &\ 2E\left(\int_0^T C_t \lambda_d N_t (1 - \tau\lambda_d N_t) dt\right) + \sigma_3^2 T \\ = &\ 2\lambda_d \int_0^T E(C_t N_t) dt - 2\lambda_d^2 \tau \int_0^T E\left(C_t N_t^2\right) dt + \sigma_3^2 T. \end{aligned} \tag{23}$$

Thus, in order to compute $E\left(C_T^2\right)$, we must first compute $E(C_t N_t)$ and $E\left(C_t N_t^2\right)$.

4.2. ODE for the mixed moments

Once again, using Ito's formula, we have

Table 3
Goodness of fit.

τ [$\times 10^{-7}$ s^{-1}]	0	3.25	5.7	8.4	12.6	16.8	25.2
$e[\%]$	0.02	0.04	0.12	0.3	0.37	0.56	1.904

$$E(N_TC_T) = E\left(\int_0^T N_t dC_t\right) + E\left(\int_0^T C_t dN_t\right) + E\left(\int_0^T dN_t dC_t\right) = \\ = E\left(\int_0^T N_t(\lambda_d N_t(1-\tau\lambda_d N_t)dt)\right) + E\left(\int_0^T C_t(-\alpha N_t + S)dt\right) - \sigma_2\sigma_3\tilde{\rho}T$$

and, denoting $\sigma^2 = \sigma_1^2 + \sigma_2^2$,

$$\begin{aligned} E\left(C_T N_T^2\right) = & E\left(\int_0^T C_t d\left(N_t^2\right)\right) + E\left(\int_0^T N_t^2 dC_t\right) + E\left(\int_0^T d\left(N_t^2\right) dC_t\right) \\ = & E\left(\int_0^T C_t\left(2N_t dN_t + \sigma^2 dt\right)\right) + E\left(\int_0^T N_t^2 dC_t\right) \\ & + E\left(\int_0^T 2N_t dN_t dC_t\right) \\ & = E\left(\int_0^T 2C_t N_t(-\alpha N_t + S)dt\right) + \sigma^2 E\left(\int_0^T C_t dt\right) \\ & + E\left(\int_0^T N_t^2 \lambda_d N_t(1-\tau\lambda_d N_t)dt\right) - \sigma_2\sigma_3\tilde{\rho}E\left(\int_0^T N_t dt\right). \end{aligned}$$

If we denote $E(N_tC_t) = Z(t)$ and $E\left(N_tC_t^2\right) = H(t)$, and use N as shorthand notation for N_{stat}, the integral equations are translated to the following set of ODE,

$$\begin{cases} \frac{dZ}{dt} = -\alpha Z(t) + SR_C t + \left(\lambda_d E\left(N^2\right) - \lambda_d^2\tau E\left(N^3\right)\right) - \lambda_d E(N) + 2\lambda_d^2\tau E\left(N^2\right) \\ \frac{dH}{dt} = -2\alpha H(t) + 2SZ(t) + \sigma^2 R_C t + \left(\lambda_d E\left(N^3\right) - \lambda_d^2\tau E\left(N^4\right)\right) \\ \quad -2E(N)\left(\lambda_d E(N) - 2\lambda_d^2\tau E\left(N^2\right)\right) \\ H(0) = Z(0) = 0 \end{cases} \tag{24}$$

Toward solving these equations, it will prove useful to write the solution as $Z(t) = Z_1(t) + Z_2(t)$ where $Z_1(t)$ satisfies

$$\frac{dZ_1}{dt} = -\alpha Z_1(t) + SR_0 t + \lambda_d E\left(N^2\right) - \lambda_d E(N); \quad Z_1(0) = 0$$

and Z_2 satisfies the equation (notice that the term SR_0t was added to the equation for Z_1, and subtracted from the equation for Z_2)

$$\frac{dZ_2}{dt} = -\alpha Z_2(t) + S(R_C - R_0)t - \lambda_d^2\tau E\left(N^3\right) + 2\lambda_d^2\tau E\left(N^2\right); \quad Z_2(0) = 0$$

The equation for Z_1, which is independent of τ, is the exact same equation for $E(DN)$ in Dubi and Atar (2018). Moreover, if $\tau = 0$ then $Z_2(t) = 0$. Thus, Z_2 can be viewed as the perturbation term in τ.

An explicit solution for $Z_1(t)$ is given by

$$Z_1(t) = \lambda_d \frac{S}{\alpha}\mathcal{C}\left(1 - e^{-\alpha t}\right) + \lambda_d\left(\frac{S}{\alpha}\right)^2 \tag{25}$$

The equation for $Z_2(t)$ is also solvable, and the explicit solution is give by

$$\begin{aligned} Z_2(t) = & -\lambda_d \frac{S}{\alpha}\left(2\tau\lambda_d \frac{S}{\alpha}\mathcal{C} - 2\tau\lambda_d(\mathcal{C} - 1)\right)\left(1 - e^{-\alpha t}\right) \\ & + \lambda_d^2 \tau \frac{S}{\alpha} E\left(N^2\right) t \end{aligned} \tag{26}$$

Next we solve the equation for $H(t)$. Before addressing this equation, we approximation it by neglecting all terms multiplied by τ. As mentioned before, this, in-effect, removes all higher powers (second and up) of the unit-less term $\tau\lambda N_t$. As we will later demonstrate (see Eq. (33)) the term $\tau\lambda N_{stat}$ approximates the average count loss. And again, the approximation will be valid as long as the count losses are not high, and may reasonably be quantified through a linear count loss. This gives

$$\frac{dH}{dt} = -2\alpha H(t) + 2SZ_1(t) + \sigma^2 R_0 t + \lambda_d E\left(N^3\right) - 2\lambda_d E(N)^2; H(0) = 0 \tag{27}$$

Eq. (27) has the general structure $y' = -2\alpha y(t) + Ae^{-\alpha t} + Bt + C$, $y(0) = 0$, admitting a solution of the form

$$y(t) = \frac{A}{\alpha}\left(e^{-\alpha t} - e^{-2\alpha t}\right) - \frac{B - 2\alpha C}{4\alpha^2}\left(1 - e^{\alpha t}\right) + \frac{B}{2\alpha}t. \tag{28}$$

Eq. (28) introduces a new mode to the system, 2α. However, if we substitute the expressions for A, B and C into it, the coefficient of $e^{-2\alpha t}$ turns out to be zero. The solution is thus given by

$$\begin{aligned} H(t) = & -2\lambda_d \frac{S^2}{\alpha^3}\mathcal{C}\left(1 - e^{-\alpha t}\right) + \left(\lambda_d\left(\frac{S}{\alpha}\right)^3 + \lambda_d\left(\frac{S}{\alpha}\right)^2 + \lambda_d\left(\frac{S}{\alpha}\right)^2\mathcal{C}\right)t \\ = & -2\lambda_d \frac{S^2}{\alpha^3}\mathcal{C}\left(1 - e^{-\alpha t}\right) + \frac{\lambda_d S}{\alpha}E\left(N^2\right)t. \end{aligned} \tag{29}$$

4.3. The Feynman-Y function of the count distribution

Having computed the mixed moments $E(N_tC_t)$ and $E\left(C_tN_t^2\right)$, we can go back to (23) and get an expression for the second moment of C_t.

To this end, note first that integration over $Z_1(t)$ (multiplied by $2\lambda_d$), as specified in Dubi and Atar (2018), gives the zero order approximation of $E\left(C^2\right)$, explicitly written as

$$\frac{\lambda_d\lambda_f\overline{\nu(\nu-1)}}{\alpha^2}\frac{\lambda_d S}{\alpha}tG(t) + \left(\frac{\lambda_d St}{\alpha}\right)^2$$

where $G(t) = 1 - \frac{1-e^{-\alpha t}}{\alpha t}$. Integration over Z_2 (again, multiplied by $2\lambda_d$) gives

$$-\frac{\lambda_d\lambda_f\overline{\nu(\nu-1)}}{\alpha^2}\frac{\lambda_d S}{\alpha}t\left(2\tau\frac{\lambda_d S}{\alpha} - 2\tau\lambda_d(\mathcal{C}-1)\right)G(t) \\ -\left(\frac{\lambda_d St}{\alpha}\right)\left(\lambda_d^2\tau E\left(N^2\right)\right)$$

and thus we have

$$\begin{aligned} 2\lambda_d\int_0^T E(C_tN_t)dt = & \frac{\lambda_d\lambda_f\overline{\nu(\nu-1)}}{\alpha^2}\frac{\lambda_d S}{\alpha}t\left(\left(1 - 2\tau\frac{\lambda_d S}{\alpha}\right) - 2\tau\lambda_d(\mathcal{C}-1)\right)G(t) \\ & + \frac{\lambda_d St}{\alpha}\left(\frac{\lambda_d S}{\alpha} - \lambda_d^2\tau E\left(N^2\right)\right)t^2 \end{aligned} \tag{30}$$

The second integral term gives

$$2\lambda_d^2\int_0^T E\left(C_tN_t^2\right)dt = \frac{\lambda_d\lambda_f\overline{\nu(\nu-1)}}{\alpha^2}\frac{\lambda_d S}{\alpha}t\left(2\tau\frac{\lambda_d S}{\alpha}\right)G(t) + \frac{\lambda_d S}{\alpha}\lambda_d^2\tau E\left(N^2\right)t^2 \tag{31}$$

We are now in a position to write a first order approximation for $Var(C_t)$, given as $E\left(C_t^2\right) - E(C_t)^2$, where $E\left(C_t^2\right)$ is given by (23) and $E(C_t)$ in (21). A straightforward calculation based on (30) and (31), shows that the coefficient of t^2 in the expression of $\mathrm{Var}(C_t)$ is proportional to τ^2. Under the assumptions mentioned earlier, this term is relatively small, and we neglect this term. As a result, we obtain the following expression:

$$Var(C_t) = \frac{\lambda_d \lambda_f \overline{\nu(\nu-1)}}{\alpha^2} \times \frac{\lambda_d S}{\alpha} t\left(\left(1 - 4\tau\frac{\lambda_d S}{\alpha}\right) - 2\tau\lambda_d(\mathcal{C}-1)\right)G(t) + \frac{\lambda_d S}{\alpha} - 3\lambda_d^2 \tau E\left(N^2\right). \quad (32)$$

Eq. (32) gives a formula for the variance, and Eq. (21) gives a formula for the mean. Hence the Feynman-Y formula can be obtained. However, from a practical point of view, this formalism suffers from a disadvantage: the term $\mathcal{C}$, which appears both in the coefficient of $G(t)$ and in $E\left(N^2\right)$ depends on λ_f, which is typically unknown. On the other hand, we may use the fact that in a typical system $|\mathcal{C}-1| \ll \frac{S}{\alpha}$. This allows us to apply two simplifying approximations: first, we neglect the term $2\tau\lambda_d(\mathcal{C}-1)$ in the coefficient of $G(t)$. Second, we neglect the term $(\mathcal{C}-1)\frac{S}{\alpha}$ in formula (2) for $E\left(N^2\right)$, and thus obtain that $E\left(N^2\right) \approx E(N)^2$. These approximations result in the following expressions:

$$E(C_t) = \frac{\lambda_d S t}{\alpha}\left(1 - \frac{\lambda_d S \tau}{\alpha}\right) = R_0(1 - R_0\tau)t \quad (33)$$

$$Var(C_t) = \frac{\lambda_d \lambda_f \overline{\nu(\nu-1)}}{\alpha^2}\frac{\lambda_d S}{\alpha} t\left(1 - 4\tau\frac{\lambda_d S}{\alpha}\right)G(t) + \frac{\lambda_d S}{\alpha}\left(1 - 3\frac{\lambda_d S \tau}{\alpha}\right) \quad (34)$$

$$Y(T) = Y_\infty G(T)\frac{1-4\tau\frac{\lambda_d S}{\alpha}}{1-\tau\frac{\lambda_d S}{\alpha}} + \frac{1-3\tau\frac{\lambda_d S}{\alpha}}{1-\tau\frac{\lambda_d S}{\alpha}} - 1 = Y_\infty \frac{1-4\tau R_0}{1-\tau R_0}\left(1 - \frac{1-e^{-\alpha T}}{\alpha T}\right) + \frac{1-3\tau R_0}{1-\tau R_0} - 1. \quad (35)$$

This formula provides a new correction to the Feynman-Y function for a system with a non-paralyzing dead time. From a practical point of view, the significance of formulas (33) and (35), other than their simplicity and novelty, is that the correction terms are expressed by two fairly easy to estimate parameters: the dead time τ and the detection rate $R_0 = \frac{\lambda_d S}{\alpha}$.

4.4. Experimental results

In the present section we aim at validating the applicability of Eq. (35) through experimental results. The data presented below was obtained from a standard noise experiment held at the MINERVE ZPR, a part of the CEA Cadarach compound. The measurement was taken during a noise measurement campaign, on June 2015. Preliminary results of the noise campaign were published in Gilad et al. (2016).

The measurement was taken at a sub-critical core with estimated reactivity of $\rho = -230$[pcm], equivalent to α value of 105[1/s]. The average count rate was 39055 CPS, recorded with a negligible dead time of less than 10^{-9}[s].

To validate the results, we have artificially imposed a non-paralyzing dead time on the measurement, simply by deleting detections for which the waiting time from the previous detection was less than the imposed dead time. This procedure was done for 6 values of τ, as described in Table 1 below.

Before implementation of formula (35), we validate formula (33) for the reduced count rate R_C, using the sampled count rate (without imposed dead time) $R_0 = 39055$. Results are shown[1] in Table 2.

Comparing the second row of Tables 1 and 2, we see that the biasing in all signals is less then 1%.

Next, we have implemented formula (35) on the sampled values of Feynman-Y plot. The values of Y_∞ and α were computed by a fit process on the signal before any dead time was induced, resulting with $\alpha = 105.7$ and $Y_\infty = 0.89$. The sampled points and the fitted curve appear in Fig. (2).

Before introducing the results we make two comments on the computability of formula (35) with previous results. First, as $T \to 0$, we see that $Y(0) = \frac{1-3R_0\tau}{1-R_0\tau} - 1 \approx -2R_0$, as suggested in Hashimoto et al. (1996). Moreover, when comparing the equation with the first order approximation in Kitamura and Fukushima (2014) (formulas (110) and (112)), the approximation is fairly similar, but not the same, due to the fact that (Kitamura and Fukushima, 2014) considers a paralyzing dead time.

The implementation of Formula (35) was done in a straightforward manner: R_c was directly sampled, and R_0 was extracted by inverting formula (33), the coefficient Y_∞ was multiplied by $\frac{1-4R_0}{1-R_0}$and the entire function was shifted (downwards) by $\frac{1-3R_0}{1-R_0}$.

Figs. 3–8 demonstrate two dead time corrections on the fitted Feynman-Y curve. The first correction (continues line) uses formula (35), and the second correction is a simple offset equal to $2R_0\tau$.

Figs. 3–6 demonstrate high correspondence between the sampled values and the analytic correction when the reduction in the CPS is in the range $1\% - 5\%$. After that, in Figs. 7 and 8, we start to see a downward bias of the corrected values. To quantify the goodness of fit for each dead time we have used a mean relative error estimation, defined by $e[\%] = \frac{1}{N}\sum_{j=1}^{N}\left|\frac{Y(T_j)-Y_j}{Y_j}\right|$. Table 3 below shows the mean relative error for all the fitted curves (including the original curve, where $\tau = 0$).

We can clearly see how the relative error grows with the dead time τ.

The fact that the quadratic approximation results with an underestimation is expected: since the function $1/(1-ax)$ is convex as a function of x, the linear approximation is smaller than the function, and thus the quadratic approximation is an over estimation of the number of detections lost.

5. Concluding remarks

In this paper, a stochastic model for the detection count rate in a sub-critical core with a detector dead time is introduced based on the SDE approach. The model assumes a single neutron group, without delayed neutrons correlations, and a non-paralyzing fixed dead time. Initially, the full model is derived. Then a first order approximation is used to explicitly compute the mean, variance and the Feynman-Y curve. Comparison with experimental results is performed, showing high compatibility with the first order approximation when the reduction in the CPS is less than 5%.

Future extensions have three natural routes. First, higher order approximations may be obtained from the full model. However, since the integration performed in the current study is complex, it is not clear at this point if the further complications associated with higher order approximations allows analogous explicit calculations. Second, one may examine the full model distribution via the associated Fokker–Planck equation. Since $E(C_t)$ tends to infinity with t, C does not follow a stationary distribution, and a parabolic partial differential equation must be considered. Finally, natural extensions include the consideration of (1) random, and (2) paralyzing dead time models. We believe that in addition to the results described in this paper, the model presented can further develop basic understanding of the effect of detection dead time in the core.

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