Lecture 1

- X and Y independent equivalent to $E[g(X)h(Y)] = Eg(X)Eh(Y)$ for all $g(\cdot), h(\cdot)$.

<u>Random vectors</u>

- The expectation of a vector

- The second moment matrix

- The covariance matrix

<u>Linear transformations</u>

$$X \longrightarrow \boxed{\quad A \quad} \longrightarrow Y = AX$$

$$EX,\ \Lambda_X \qquad\qquad EY = AEX,\ \Lambda_Y = A\Lambda_X A^T$$

- The second moment matrix is always nonnegative definite.

<u>Characteristic function</u>

$$\varphi_X(\nu) = E[e^{i\nu^T X}]$$

- Related to Fourier transform

- $\varphi_X(\nu) = \prod_i \varphi_{X_i}(\nu_i)$ if and only if X_i are mutually independent.

$$X \longrightarrow \boxed{\quad A \quad} \longrightarrow Y = AX$$

$$\varphi_X(\nu) \qquad\qquad \varphi_Y(\nu) = \varphi_X(A^T\nu)$$

<u>The Gaussian distribution</u>

Lecture 2

Gaussian random vectors

- Linear transformation of a Gaussian random vector is a Gaussian random vector.

- Formula for the characteristic function.

- Formula for the density function.

- Diagonal covariance matrix implies mutual independence of the components.

- One can find nonsingular linear transformation that makes the components independent.

Conditional expectation

- Definitions.

- Smoothing theorem.